

Comparing Linear & Exponential Functions

PROBLEM 1 Let's Build a Formula for Simple Interest



Simple Interest: Interest that is calculated using the original (principle) balance each term.

Annual Percentage Rate (APR): The interest rate used to calculate interest once per year. It is always written as a decimal when used in a calculation.



- Suppose that Nico deposits \$1000 into an account that earns 5% simple interest each year. Complete the table to show Nico's account balance after each year. Show your work. The first three rows have been completed for you.

Value in the time column is 1 less than the corresponding term number n . This is because a sequence always begins with the first term at $n = 1$, but the problem situation begins at time = 0.



Term Number (n)	Time (years)	Interest Earned (dollars)	Account Balance (dollars)
1	0	0	1000
2	1	$1000(0.05) = 50$	$1000 + 1000(0.05) = 1050$
3	2	$1000(0.05) = 50$	$1050 + 1000(0.05) = 1100$
4	3	$1000(.05) = 50$	$1100 + 50 = 1150$
5	4	$1000(.05) = 50$	$1150 + 50 = 1200$
6	5	$1000(.05) = 50$	$1200 + 50 = 1250$

+50
+50
+50
+50
+50



- Does the account balance form an Arithmetic or Geometric sequence? How do you know? Find the common difference or ratio.

Arithmetic. the balance goes up at a constant rate of \$50 per year.

$$d = 50$$



Writing and Solving Simple Interest Equations

In a simple interest account, the interest you earn at the end of each year is a percent of the original principal. The pattern of the account balance year after year creates an arithmetic sequence because the same amount is added to each term to create the next term in the sequence. The sequence can be written as a function in terms of time: $P(t) = P_0 + (P_0 \cdot r)t$, where t is time, $P(t)$ is the total balance after t years, P_0 is the original principal, and r is the interest rate written as a decimal.

1. An account earns 3% simple interest on a deposit of \$500.

$$P_0 = 500$$

$$r = .03$$

- a. Write a function to represent this simple interest situation.

$$P(t) = 500 + 500(.03)t$$

$$P(t) = 500 + 15t$$

- b. How much money is in the account after 8 years? Show how you got your answer.

$$P(8) = 500 + 15(8)$$

$$= \$620.00$$

- c. How many years it will take for the account to have \$725 in it? Show how you got your answer.

$$P(t) = 500 + 15t$$

$$725 = 500 + 15t$$

$$\begin{array}{r} 725 \\ -500 \\ \hline 225 = 15t \end{array}$$

↑
output

$$t = 15 \text{ years}$$

2. Perry deposits \$175 into a simple interest account. The interest rate for the account is 4.25%.

- a. Complete the table for the simple interest.

	Time	Simple Interest Balance
Units	years	dollars
Expression	t	$175 + 7.44t$
	0	175
	1	182.44
	2	189.88
	3	197.32
	10	249.40

$$P_0 = 175$$

$$r = 0.0425$$

$$P_0 \cdot r = 175(.0425)$$

$$= 7.44$$

- b. How much money will Perry have in her account after 18 years?

$$P(t) = 175 + 7.44(18) = \$308.92$$

PROBLEM 2 Now Let's Build a Formula for Compound Interest

Compound Interest: Interest that is calculated using the balance of the previous term.



1. Suppose that Raul deposits \$1000 into an account that earns 5% compound interest each year. Complete the table to show Raul's account balance after each year. Show your work. The first three rows have been completed for you.

Term Number (n)	Time (years)	Interest Earned (dollars)	Account Balance (dollars)
1	0	0	1000
2	1	$1000(0.05) = \$50$	$1000 + 1000(0.05) = 1050$
3	2	$1050(0.05) = 52.5$	$1050 + 1050(0.05) = 1102.5$
4	3	$1102.5(.05) = 55.125$	$1102.5 + 55.125 = 1157.625$
5	4	$1157.625(.05) = 57.88125$	$1157.625 + 57.88125 = 1215.50625$
6	5	$1215.50625(.05) = 60.7753125$	$1215.50625 + 60.7753125 = 1276.2815625$



2. Is the account balance an Arithmetic or Geometric sequence? How do you know? Find the common difference or ratio.

Geometric. there is a common ratio between consecutive terms.

$$\frac{1050}{1000} = 1.05$$

$$\frac{1102.5}{1050} = 1.05$$

$$r = 1.05$$

$$\frac{1157.625}{1102.5} = 1.05$$



Writing and Solving Compound Interest Equations

In a compound interest account, the interest you earn at the end of each year is a percent of the account balance at the beginning of the year. The pattern of the account balance year after year creates a geometric sequence because the same amount is multiplied by each term in the sequence to produce the next. The sequence can be written as a function in terms of time: $P(t) = P_0 \cdot (1 + r)^t$, where t is time, $P(t)$ is the total balance after t years, P_0 is the initial deposit amount, and r is the interest rate each year, written as a decimal.

1. An account earns 3% compound interest on a deposit of \$500.

$$P_0 = 500$$

$$r = .03$$

- a. Write a function to represent this ~~simple~~^{compound} interest situation.

$$P(t) = 500(1 + .03)^t$$

$$P(t) = 500(1.03)^t$$

- b. How much money is in the account after 8 years? Show how you got your answer.

$$P(8) = 500(1.03)^8$$

$$= \$633.39$$

- c. How many years it will take for the account to have \$725 in it? Show how you got your answer.

$$P(t) = 500(1.03)^t$$

$$\frac{725}{500} = \frac{500(1.03)^t}{500}$$

$$1.45 = (1.03)^t$$

12 < t < 13 ← "guess + check"

$$t \approx 12.5 \text{ years}$$

2.

Suppose that your family deposited \$10,000 in an interest bearing account for your college fund that earns 4% simple interest each year and a friend's family deposited \$10,000 in an interest bearing account for their child's college fund that earns 4% compound interest each year.

Use the simple and compound interest formulas to complete the table and round the values in the table to the nearest cent.

	Time	Simple Interest Balance	Compound Interest Balance
Units	years	dollars	dollars
Expression	t	$10000 + 400t$	$10000(1.04)^t$
	0	10,000	10,000
	1	10,400	10,400
	2	10,800	10,816
	3	11,200	11,248.64
	10	14,000	14,802.44

$$P_0 = 10,000$$

$$r = .04$$

$$P_0 \cdot r = 400$$

$$1 + r = 1.04$$

How much money will you and your friend have in the college funds when you each turn 18 years old?

$$10,000 + 400(18)$$

$$= \$17,200$$

$$10,000(1.04)^{18}$$

$$= \$20,258.17$$